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Spatial Reasoning with Uncertain Data Using Stochastic Relaxation

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Abstract

Spatial Reasoning often uses sensor data as input. Since sensor data normally have distortions problems of uncertainty and possible contradictions arise. Classical AI inference techniques have difficulties in dealing with uncertainty and contradictions. Thus our approch is to use soft computing techniques for our domain. We augmented the simulated annealing approch by composition tables as ternary constraints and relation sets as variables. Some constraints model physical laws which have strong confidence. Others measure data with much lower confidence. Therefore we also have to integrate soft and hard constraints.

Keywords: approximate reasoning, hybrid systems, spatial reasoning

1 Introduction

Sensor data are often used as input by spatial reasoning. Examples are the analysis of satelite images in geographical information systems, the speech recognition in natural language interfaces and landmark identification in robotic navigation. However, sensor data may include contradictions. Classical inference techniques as used for qualitative reasoning , e.g., constraint propagation [Allen, 1983, Hernández, 1994] or the application of theorem provers for predicate calculus [Cohn, 1997], are unable to produce inferences if contradictions occur. In this paper we would therefore like to introduce inference techniques which are still able to draw inferences and offer partial solutions if inconsistencies occur.

The starting point for our approach is the *conceptual neighbourhood* property [Freksa, 1992]. The conceptual neighbourhood describes transitions between qualitative spatial and temporal relations. A continuous increase of measurement errors produces a transition to adjacent relations (wrt. the conceptual neighbourhood structure). Conceptual neighbourhood structures are independent of applications and available for most spatial calculi. Therefore, this procedure is a general method to deal with uncertain and inconsistent spatial facts.

A conceptual framework for inference machines which employs this view is the partial constraint satisfaction technique [Freuder and Wallace, 1992]. Partial constraint satisfaction means not to satisfy all constraints if the information is inconsistent. It is then important to validate the solutions. The validation of the solutions with respect to violated constraints is performed with a metric. For qualitative spatial reasoning calculi the conceptual neighbourhood can be used to express different degrees of constraint violations. This extended model generates partial solutions despite the occurence of contradictions in the input sensor data.

Partial constraint satisfaction can be realized with graph search [Freuder and Wallace, 1992]. However, one should expect combinatoric explosion to appear. With this contribution we pursued the strategy to augment simulated annealing [Reeves, 1993, Aarts and Korst, 1990] to our problem.

2 Conceptual neigbourhoods and spatial reasoning

Qualitative spatial calculi usually deal with elementary objects (e.g., positions, directions, regions) and qualitative relations between them (e.g., "adjacent", "on the left of", "included in"). If the objects g_i, g_j and g_k have the relations $g_i\,\mathrm{R1}\,g_j$ and $g_j\,\mathrm{R2}\,g_k$, then with a composition table one can infer a relation R3 between g_i and g_k : $g_i\,\mathrm{R3}\,g_k$.

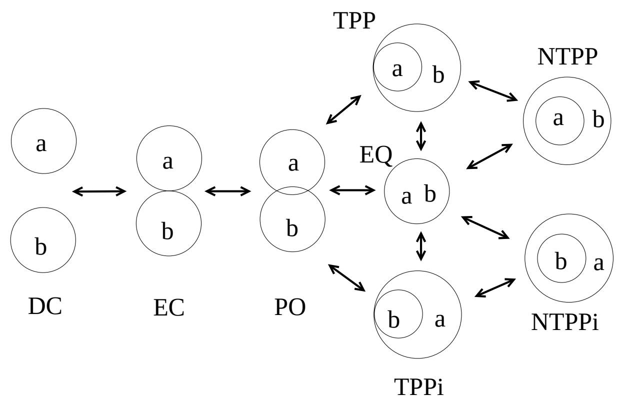

Figure 1: Conceptual neigbourhoods in a topological region calculus

Conceptual neigbourhoods then represent continuous transitions between relations. We use the RCC-8 calculus [Cohn, 1997] here as an example for a spatial calculus. The RCC-8 theory models topological relations between regions (see figure 1). The RCC-8 calculus has the set C of possible qualitative relations between regions:

$$C = \{\mathrm{DC}, \mathrm{EC}, \mathrm{PO}, \mathrm{EQ}, \mathrm{TPP}, \mathrm{NTPP}, \mathrm{TPPi}, \mathrm{NTPPi}\} \tag{1}$$

For example, the conclusion from $g_i\,\mathrm{NTPP}\,g_j$ and $g_j\,\mathrm{NTPP}\,g_k$ to $g_i\,\mathrm{NTPP}\,g_k$ is a valid inference in RCC-8. The inferences of all these pairs of relations are summarized in a composition table (see [Cui et al., 1993]) and are the central component of the

RCC-8 calculus. The composition table is represented as a function $\mathrm{f} : C \times C \mapsto 2^C$ (for example $\mathrm{f}(\textsc{ntpp}, \textsc{ntpp}) = \{\textsc{ntpp}\}$).

In a simple geometrical model of the RCC-theory the regions are mapped as circles. Then the continuous transitions between qualitative relations are the continuous modifications from center and radius of the circle regions. This corresponds to the conceptual neighbourhoods as can be seen in figure 1. A more general model for the RCC-theory are closed polygons. Therefore, a continuous transformation would be the displacement of polygon vertices. This more elaborate model is used in visual object recognition [Moratz, 1997]. The conceptual neighbourhood is represented as function
$\mathrm{d} : C \times 2^C \mapsto N$ (for example $\mathrm{d}(\textsc{po}, \{\textsc{tppi}, \textsc{ntpp}\}) = 1$). The function d then is a distance measure for qualitative spatial relations.

3 Simulated annealing for spatial reasoning

With simulated annealing constraint problems can be solved. Such a problem consists of a set of variables with finite domains and a set of constraints each of which refers to a subset of variables. The constraints model the reciprocal compatibilities of variables. In particular binary constraint problems can be regarded as nets with variables as nodes and constraints as edges [Hinton and Sejnowski, 1986]. An energy function assigns an energy value to a net state, i.e., to a variable assignment. Low energy corresponds to a state of high reciprocal compatibility, i.e. in which there is little violation of constraints. Thus the algorithm searches for a variable assignment with an energy as small as possible. In classical simulated annealing the variables have a binary domain. Then the nodes can be interpreted as artificial neurons. Usually one uses the following energy function:

$$E = -\sum_{i<j} w_{ij}\, s_i\, s_j + \sum_i \theta_i\, s_i \tag{2}$$

In this function s_i is the value of the i-th variable (i.e., the activation of the i-th node), w_{ij} is the constraint value between the i-th and the j-th variable (i.e., the connection strength of the corresponding edge). The threshold θ of each variable represents a unary constraint. Its effect is the same as that of an edge with the same weight to a node with constant value one. If the value of the variable changes the net will change from state α into state β. The probability for this transition follows a Boltzmann-distribution and is therefore defined by the energy difference of both states:

$$\frac{P_\alpha}{P_\beta} = e^{-(E_\alpha - E_\beta)/T} \tag{3}$$

In this formula T is the temperature parameter that regulates the strength of the tendency towards low-energy states. In a low energy state there is a greater risk to get caught in local minima, however we can faster reach the minima. A reasonable approach is to start with the higher temperatures and to slowly decrease.

With a direct extension of the simulated annealing technique this standard model can be applied to qualitative spatial reasoning calculi. Then, the domain of the variables is the set C of all qualitative relations of the spatial calculus. The composition table is used to create ternary constraints between relations. The measured data for the observed relations are modeled as unary constraints in our approach.

The node s_{ij} represents the relation between the regions g_i and g_j. Each node s_{ij} has a relation as value, $s_{ij} \in C$. The measurement m_{ij} is a subset of C, $m_{ij} \subseteq C$. In case of no measured information, then $m_{ij} = C$. A ternary composition-constraint has the form $\mathrm{d}(s_{ik}, \mathrm{f}(s_{ij}, s_{jk})) = 0$. This means that the relations s_{ij}, s_{jk} and s_{ik} are compatible, if the resulting relation s_{ik} can be covered by the composition of s_{ij} and s_{jk}. As the energy function E to be minimized we use the following formula:

$$E = \lambda \cdot \sum_{i<j} \mathrm{d}(s_{ij}, m_{ij}) + (1-\lambda) \cdot \sum_{i<j<k} \mathrm{d}(s_{ik}, \mathrm{f}(s_{ij}, s_{jk})) \tag{4}$$

In this function λ is the weight between measurement-constraints and composition-constraints.

Then, the algorithm selects a node s_{ij} at random and assigns him a new value. The probability for a decision of a certain relation value $R1$ depends on the local energy $E_{ij}(R1)$. This local energy is the local contribution of the node s_{ij} to the total energy, $\sum_{i<j} E_{ij} = E$. From this follows:

$$\begin{aligned} E_{ij}(R1) &= \lambda \cdot \mathrm{d}(R1, m_{ij}) + \frac{1-\lambda}{3} \cdot \left(\sum_{k=1}^{i-1} \mathrm{d}(s_{kj}, \mathrm{f}(s_{ki}, R1)) \right. \\ &\left. + \sum_{k=i+1}^{j-1} \mathrm{d}(R1, \mathrm{f}(s_{ik}, s_{kj})) + \sum_{k=j+1}^{n} \mathrm{d}(s_{ik}, \mathrm{f}(R1, s_{jk})) \right) \end{aligned} \tag{5}$$

From the Boltzmann-distribution (see equation 3) we can derive the stochastic decision-rule:

$$P(s_{ij} = R1) = \frac{e^{-E_{ij}(R1)/T}}{\sum_{r \in C} e^{-E_{ij}(r)/T}} \tag{6}$$

In the next section a small sample problem shows how to select adequate values of T and λ.

4 A sample problem

A small problem with a contradiction in the input data is shown on figure 2. It consists of four regions with the $\binom{4}{2}$ initial relations: g_1 NTPPi g_2, g_1 EC g_3, g_1 DC g_4, g_2 DC g_3, g_2 EC g_4, g_3 NTPPi g_4. The problem is to find a consistent interpretation

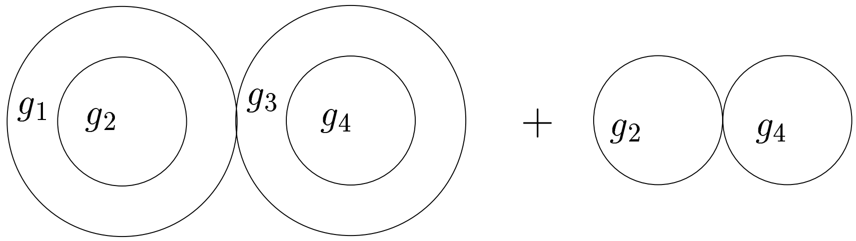

Figure 2: A problem with contradictory input data

(realizable configuration) which changes the inital data as little as possible. The constraint network has six nodes corresponding to the six relations. The initial data serve as six unary constraints. There are $\binom{4}{3} = 4$ ternary composition constraints: $\mathrm{d}(s_{13}, \mathrm{f}(s_{12}, s_{23})) = 0$, $\mathrm{d}(s_{14}, \mathrm{f}(s_{12}, s_{24})) = 0$, $\mathrm{d}(s_{14}, \mathrm{f}(s_{13}, s_{34})) = 0$, $\mathrm{d}(s_{24}, \mathrm{f}(s_{23}, s_{34})) = 0$. The state which satisfies all initial input data (e.g. $s_i j = m_i j$) would violate two constraints:

$\mathrm{d}(s_{14}, \mathrm{f}(s_{12}, s_{24})) = \mathrm{d}(\textsc{dc}, \mathrm{f}(\textsc{ntppi}, , \textsc{ec},)) = \mathrm{d}(\textsc{dc}, \{\textsc{po}, \textsc{tddi}, \textsc{ntppi}, \}) = 2$

$\mathrm{d}(s_{24}, \mathrm{f}(s_{23}, s_{34})) = \mathrm{d}(\textsc{ec}, \mathrm{f}(\textsc{dc}, , \textsc{ntppi},)) = \mathrm{d}(\textsc{ec}, \{\textsc{dc}\}) = 1$. One cycle of the algorithm consists of updating each relation in a random sequence. Begining with $T = 1$ and $\lambda = 1$ and setting in the nth-step $T = 0.9^{n-1}$ and $\lambda = 0.9^n + 0.4$ the system converges typically after 5 to 10 cycles to the following solution: $g_1\,\textsc{ntppi}\,g_2$, $g_1\,\textsc{ec}\,g_3$, $g_1\,\textsc{dc}\,g_4$, $g_2\,\textsc{dc}\,g_3$, $g_2\,\textsc{dc}\,g_4$, $g_3\,\textsc{ntppi}\,g_4$. This solution satisfies all hard constraints (physically motivated constraints) and violates the soft constraints (measures with possible errors) as little as possible.

5 Conclusion

With our approach we want to establish a method for spatial reasoning with inconsistent input data using soft computing techniques. Using conceptual neigbourhoods representing continuous transitions between relations we had a metric for partial solutions. We augmented the simulated annealing approch by composition tables as ternary constraints and relation sets as variables. The integration of soft initial constraints and hard constraints which model physical laws ist done by introducing a special parameter. Our approach works well for small applications. The next step will be to examine the scaling behaviour of our algorithm using more complex tasks.

Our approach can be used to meet other requirements. A similar situation to the application of uncertain data is the existence of a contradiction-free solution, which can only be obtained with a disproportional big effort (e.g., exponential costs). Then, it might be better to be content with a different solution that is suboptimal, but can be calculated effectively. Also the framework of anytime algorithms (e.g., the ability to retrieve partial solutions at an externally given time) would be an application field for our approach. The spatial domain is not the only possible application area. At least for the temporal domain [Allen, 1983] the same technique can by applied.

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