Shape Nouns and Shape Concepts: A Geometry for 'Corner'

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Abstract. This paper investigates geometric and ontological aspects of shape concepts underlying the semantics of nouns. Considering the German shape nouns *Ecke* and *Knick* (*corner* and *kink*) we offer a geometric framework to characterize substantial aspects of shape based on features of the object's boundary. Using the axiomatic method, we develop a geometric system, called 'planar shape geometry', enriching the basic inventory of ordering geometry by shape curves. The geometric characterization is not sufficient to decide which are the referents of the nouns *Ecke* and *Knick* among the entities involved in the spatial constellation. Different tests using the German topological prepositions *in* and *an* (*in* and *at*) are employed to bring forth this decision for the case of *Ecke*. Since these tests do not give uniform evidence in favor of one solution, we have to conclude that *Ecke* is flexible in selecting the referent and the characterizations discussed reflect its meaning spectrum.

1 Introduction

1.1 Language and Space: The Role of Conceptual Representations

Human behavior is anchored in space. The processing of spatial information has a central position for human cognition, since it subsumes information about spatial properties of the entities in our environment, about spatial constellations in our surroundings, and about the spatial properties and relations of our bodies with respect to these surroundings. Spatial information is essential for the recognition of objects and events by different sensory channels, i.e., in visual, haptic or auditory perception. Locomotion and body movement are based on such information as well.

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Beyond perception and motor action, some higher cognitive capacities such as memory, problem solving, and planning are based on spatial representations (cf. Eilan, McCarthy & Brewer, 1993). Not only communication about space using natural language involves spatial language, but the systematic use of spatial terms in other domains is a general feature of human communication. This suggests that abstract spatial concepts play an important role in non-spatial domains (see Mandler, 1996; Habel & Eschenbach, 1997).

The study of the relationship of language and space is a major field within cognitive science (see, e.g., Bloom et al., 1996). A widely "consensually accepted framework within which the relations between language and space have been considered" has been established (Peterson et al., 1996, p. 553), namely that conceptual representations are an interface between language and spatial cognition. This holds despite terminological variations and different demarcations between the modules involved among distinct branches of the research on language and space (see Peterson et al., 1996; Habel, 1990; Jackendoff, 1991; Landau & Jackendoff, 1993; Bierwisch & Lang, 1989; Bierwisch, 1996).

The human cognitive system has two major ways of gaining information about space: On the one hand, via perception and proprioception, and on the other hand, via communication, especially using natural language. Conceptual representations are fundamental for the relation between language and space since linguistic and spatial conceptual representations constitute the linguistic-spatial interface. Conceptual representations encode meaning independent from any particular language. They "refer not to the real world or to possible worlds, but rather to the world *as we conceptualize it*" (Jackendoff, 1996, p. 5). Leaving aside details of the interface, studying spatial terms leads to further insights about the deeper levels of conceptual representations (see Jackendoff, 1996; Peterson et al., 1996, p. 555).

The units of the conceptual structure are called 'concepts' in the following. This usage of 'concept' is a generalization of the standard usage of 'concept' in psychology, which focuses on the type called 'nominal concept' by Miller (1978). The restriction of the standard view is based on taking concepts as the means for categorization (Smith, 1995, p. 3) and categorization as the placement of objects in classes only. But since humans are able to categorize constellations of objects or object parts and to recognize one object as being the same as an object encountered before, it seems suitable to extend the notion of 'concept'. According to Miller (1978) we assume that the conceptual structure includes different types of concepts, e.g. concepts for relations, properties and objects. In this sense, touching, betweenness or corner are relational concepts, while sphere, round or angular are nominal concepts. In contrast to this, our concepts of the moon or of Ray Jackendoff are object concepts.

Since conceptual structures are independent from individual languages, the correspondence between lexemes and concepts is usually not of the one-to-one type. Hence, it is important to distinguish between lexemes and their conceptual counterparts, even if they are referred to by the same string; we consider this by typographical differences, e.g. the lexeme *corner* vs. the concept corner.

The structure of representations relating language and space (see Fig. 1) reflects the tasks to be performed in the analysis of spatial language and spatial cognition: Lexemes (1) constitute the starting point of the analysis. Examples are spatial prepositions as *in* or *behind*, verbs as *enter* or nouns as *corner*. A systematic variation of different combinations of lexemes determines their conceptual similarities and differences. In addition, the applicability and interpretability of simple and complex spatial expressions can be tested with regard to different spatial situations.

A core idea of conceptual semantics is that the lexical entry of a spatial term specifies a concept (2) that serves as a representation of some spatial constellation in the external world. The geometric characterization (3) of spatial concepts (2) is the mathematical description of empirical considerations on conceptual semantics. The primary goal of the formal characterization is to determine some candidates for an inventory of basic spatial concepts, which can be seen as building blocks for a system of spatial concepts.

Lexicon (1)		Conceptual Structure (2)		Spatial (Geometric) Representations (3)
lexemes	→	concepts	←	geometric characterizations

Fig. 1. Types of entities used in the analysis of spatial terms of natural language

To develop a mathematical, e.g. topological or geometric, framework we employ the axiomatic method.[1] Instead of defining the basic concepts, an axiomatic system constitutes a system of constraints that determines the properties of these concepts by specifying their interrelations. Hence, axiomatic specifications of spatial properties and relations provide exact characterizations. Different axiomatic systems—of one given set of spatial relations—can be compared as to how restrictive they are.

In section 3 we develop a geometric framework, providing a formal basis to model empirical linguistic findings. In general, the axiomatic systems we propose for groups of spatial concepts are motivated by the analysis of natural language. The generality of the theories developed using the axiomatic method is sometimes seen as a disadvantage, since they cannot be restricted to the one (intended) model the analysis is based on. But in the context of the analysis of natural language expressions, this property turns out to be an advantage again, since the variability of expressions with respect to their domain of application is a well known feature of natural languages (cf. Habel & Eschenbach, 1997).

1.2 Shape Concepts: Axes and Boundaries

The importance of shape information is emphasized in many areas of cognitive science, especially with respect to visual perception and categorization.[2] In spite of its importance, there is a lack in lucidity what shape is. For instance, there is no explicit characterization of shape, but only an informal agreement among scientists in the field of visual perception. This agreement has been the basis of many approaches to understand visual perception, which are described in Pinker's (1984) overview of central issues in visual cognition. Starting with the Marr-Nishihara theory (1978), the

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[1] Starting in mathematics and physics the axiomatic method has spread to other disciplines (see, Henkin et al., 1959, compare also the detailed discussion by Luce et al., 1990).

[2] Rosch et al. (1976) point out that shape belongs to the key properties that structure 'basic level categories', e.g. those which are named by count nouns like *apple*, *hat*, *cup*.

representation of shape is mainly based on spatial symmetry and object axes, which play a central role both on the level of the whole object and that of the components.

In linguistic semantics the approach of Bierwisch and Lang (1989) on dimensional adjectives, e.g. *long* or *high*, also emphasizes the role of axes for shape descriptions. Object schemata, which serve as explanatory basis for the semantics of these terms (see Lang, 1989), are conceptual representations of objects that specify their main axes as well as the proportions between the axes; this information is matched against a corresponding slot in the lexical entry of dimensional adjectives. The interdependence of shape and axes as seen by Bierwisch and Lang is expressed in Bierwisch's specification of shape of objects as "the proportional metrical characteristics of objects and their parts with respect to their conceptually relevant axes or dimensions (3-D models)" (Bierwisch, 1996, p. 47). [3]

The other core notion for characterizing shape is 'boundary'. Perception research lays emphasis on the use of boundaries in decomposing objects into their parts, especially on describing rules for the detection of part boundaries, e.g. based on notions as "concavities" of "concave regions" or the "minima rule" (see Marr & Nishihara, 1978; Biederman, 1987; Hoffman & Richards, 1984; Hoffman & Singh, 1997). Biederman's (1987) recognition-by-component (RBC) theory supplies an inventory of 36 geons (generalized cones) that are derived from properties of edges in a planar image. These properties include curvature, collinearity, symmetry, parallelism and cotermination. The resulting geons can be seen as building blocks for the specification of objects by their components (cf. Biederman, 1995).

The RBC-theory and the idea of geons have been influential in linguistic and psycholinguistic approaches to shape. For example, Landau and her colleagues assume the same basic principles of characterizing 'shape' by the arrangement of object parts as the object recognition theories described above. On this basis, Landau, Smith, and Jones (1988) show that children and adults use similarity in shape much more than similarity in size or texture in generalizing a novel count noun to new objects.

Characteristics of boundaries seem to play an important role also on the conceptual level: Landau et al. (1992) report a series of experiments that emphasize the influence of different types of boundaries (angular vs. curved edges) as cues for the acceptability of shape transformations. These experiments show that, despite differences in the global shape, objects with curved boundaries are more easily grouped together than objects with angular boundaries. They suggest that angular boundaries in contrast to curved boundaries are seen as evidence for rigidity.

In the present paper we focus on German lexemes—namely, *Ecke (corner)* and *Knick (kink)*[4]—that involve properties of the boundary of an object. Starting with an informal and general characterization of the spatial constellation underlying the uses

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[3] "Dimension" as used by Bierwisch & Lang (1989) is widely identified with the concept of axis, which is taken to be an internal bounded straight line. Hence, it is possible to compare axes with respect to their length and thus to use axes for characterizing proportion.

[4] The analysis reported was carried out with respect to the German lexemes. Note that their English counterparts, which we use in the text for readability, differ in some respects. E.g., the English noun *corner* is used for parts of the mouth or the eye. In contrast, the German noun *Ecke* cannot be used to refer to such parts. Instead *Winkel (angle*) is used. This observation suggests that *corner* is not as clearly restricted to objects with straight edges as *Ecke* is.

of these nouns, we present in section 3 a geometric framework that allows us to specify the spatial aspects of their semantics. The spatial constellation specified is neutral with respect to the question of what is the referent of a phrase like *a corner of the carpet*. The referent might be just a point, a part of the boundary of the carpet, or a part of the carpet itself. The details of these possibilities are given in section 4. Which alternative is the most appropriate one will be discussed in section 5 in connection with the more detailed linguistic analysis.

2 A First Glance at Corners and Kinks

The most obvious similarity between corners and kinks is that they possess vertices. Still, the terms *Ecke* and *Knick* are mostly mutually exclusive.

2.1 Dimensionality

One important factor for differentiating corner and kink can be grasped by the idea of 'dimensionality'. Since we are not aiming at a general account on dimensionality, we will leave it at an informal characterization by giving examples of some mathematical entities: points are zero-dimensional, line segments are linear or one-dimensional, squares are planar or two-dimensional and cubes are three-dimensional. This exemplification meets the interpretation of 'dimensionality' mostly used by linguists, such as Bierwisch & Lang (1989) or Jackendoff (1991), who characterize dimensionality via the number of orthogonal axes of the object. This traditional view has been replaced in modern geometry and topology using recursive or inductive definition schemes that explain the dimension of a space (or a subspace) by the dimension of its boundary.[5]

The differences in applicability of *Ecke* and *Knick* correspond to differences in the dimensionality of the objects. *Knick* concerns a property of paths, sticks, or other objects with a linear appearance.

(1) a. Der Weg hat einen Knick.	(The path has a kink.)
b. Der Metallstab hat einen Knick.	(The metal stick has a kink.)
(2) ?Der Metallstab hat eine Ecke.	(The metal stick has a corner.)

In both cases in (1), it is impossible to replace the noun *Knick* by *Ecke.* A metal stick that exhibits a vertex where two straight parts of the stick meet can be described using (1.b). In contrast, (2) is not appropriate to describe the same situation. Since this is independent from any contextual influence, there seems to be a semantic or conceptual conflict between *Stab* (*stick*) and *Ecke*, which prohibits their combination. However, the contour itself is not sufficient to judge whether *Knick* or *Ecke* is the

[5] See Kline (1972, p. 1161f) on the history of these mathematical ideas. Although Jackendoff (1991, p. 32) also formulates the idea that "a boundary has one dimension fewer than what it bounds", he does not take the step of separating dimensionality and axes. From our point of view, this separation is necessary to analyze boundary-shape concepts independently from axes-shape concepts.

appropriate lexeme. To clarify this, we consider the example of a metal stick having four kinks such that it bounds an area (Fig. 2). This situation can be described by (3.a), (3.b) and by (3.c), but not using (3.d).

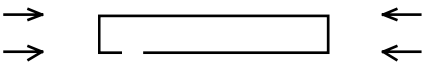

Fig. 2. Depiction of a metal stick with four (highlighted) kinks

(3) a. Der Metallstab hat vier Knicke.	(The metal stick has four kinks.)
b. Die Fläche hat vier Ecken.	(The area has four corners.)
c. Der Bilderrahmen hat vier Ecken.	(The picture frame has four corners.)
d. ?Der Metallstab hat vier Ecken.	(The metal stick has four corners.)

The differences between the linguistic descriptions in (3) reflect the different conceptualizations of the entities in question. While the metal stick is considered as a linear object in (3.a), the area in (3.b) and the picture frame in (3.c) are considered as planar objects. Thus, the conceptualized dimension of an object in combination with the shape properties determines the lexical decision. Since conceptualization, and therefore mental representation, is involved, this difference can be seen as the difference between the concepts kink and corner.

Kink and corner differ with respect to the conceptualized dimensionality of the underlying objects: Kink requires linear objects and corner requires at least planar objects. Although a metal stick can induce a planar region as in Fig. 2, we explicitly have to refer to the induced region and not to the object inducing the region, if we want to refer to its corners, cf. (3.b) vs. (3.d).

Thus, although sticks are extended with respect to three dimensions, and paths are at least planar, their linear conceptualizations are the basis for the constitution of a kink-constellation as expressed in (1). Sticks and paths can be considered as linear objects due to their prominent elongation axes. The characterizations of kink in section 4.2 reflects this observation.

The analysis of *Ecke* in the present paper focuses on planar examples like *corner of a window* or *corner of a carpet*. Corners of three-dimensional objects—e.g. corners of rooms, houses, cupboards, etc.—can be classified depending on whether the object is conceptualized as planar or three-dimensional: *the carpet in the corner of the room* needs only the consideration of the planar outline of the room, while *the spider net in the corner of the room* can geometrically best be described based on three dimensions. The restriction to the planar case is due to our observation that the basic points we want to discuss with respect to the linguistic behavior of *Ecke* and the geometric characterization of corner do not depend on the distinction between two- and three-dimensional corners.

Dimensionality is an important aspect in investigating the ontological essence of corners also in another way. In section 5 we discuss whether corners are vertex points, parts of the boundary (i.e., linear), enclosed regions (i.e., planar), or parts of objects. This line of investigation is in principle independent of the question whether the objects they are corners of are conceptualized as linear or planar, since corners or kinks need not be of the same dimension as the object they belong to.

2.2 Sharp Concepts in Flexible Use

In the remainder of this section we discuss geometric aspects of corner to motivate the geometric analysis given in section 4. The general points hold for kink and other shape nouns as well. The more detailed discussion of the lexeme *corner* in section 5 focuses on aspects of its meaning that cannot be determined by geometric means.

Ideal corners, like the corners of a square or a triangle, exhibit a vertex at which two straight parts of the object's boundary meet. But the applicability of the lexeme *corner* is vague with respect to at least three independent aspects.

- Vagueness in size: Corners can, depending on the context, reach far to the middle of an object, or can be a very small part close to the vertex.
- Idealization of the vertex: Corners of material entities do not have to exhibit a proper vertex at which two straight lines meet, e.g., corners of tables are not sharp-edged in most cases (see Fig. 3.b).
- Idealization of the edges: The two edges meeting at the corner need not be straight as line segments in geometry. The boundary of a material object can be wavy or saw-toothed up to a certain degree, like the boundary of a stamp (see Fig. 3.c, d).

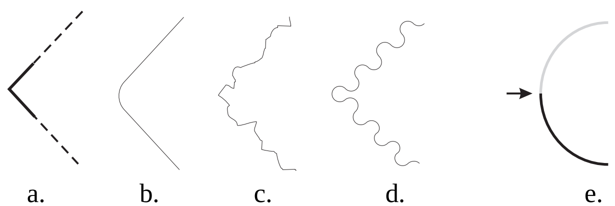

Fig. 3. Four depictions of corners in contrast to smoothly meeting boundary parts

The review of these aspects of vagueness leads to the question, whether we have to consider all geometric deviations that real-world referents of the count noun *Ecke* show in the geometric characterization of corner. First of all, the vagueness in size seems intrinsic to the concept of corner. The extent of corners cannot be given sharply with respect to other parts of the object (this is indicated by the dashed lines in Fig. 3.a). Thus, the formal characterization of corner shall reflect this vagueness.

In contrast to this, the vagueness in regard to the sharpness of the vertex and the straightness of the edges concerns the relation between corner-instances in the real world and the concept corner or the lexeme *Ecke*, respectively. It regards the question how well reality and geometry fit together. Akin to Miller & Johnson-Laird (1976), we assume a definite conception for corner and flexible mappings of the real world to the conceptual level that can make sense of the world as it is perceived, i.e., that are able to handle the wide variety of instances of members of a category humans are confronted with in perception.

We want to exemplify this flexibility with the case of the corners of stamps (see Fig. 3.d). In order to assign corners to a stamp, it can be ignored that its border is saw-toothed. The detailed geometric characteristics of the border of the stamp are not relevant for its global shape. It is sufficient to conceive of this border as consisting of straight line segments that are induced from the arrangement of the teeth of the stamp. Thus, the attribution of having corners is based on the possibility to cognitively induce straight edges from global characteristics of the object. Hence, the formal characterization of corner developed in section 4 sticks to straight edges.

Kink on the other hand does not need this requirement: If a kink is ascribed to a metal stick as in (1.b), this does not demand that the adjacent parts of the vertex have to be straight. Instead they may be curved. To reflect this distinction between kink and corner we develop a framework that distinguishes straight from curved lines.

Another essential component of this concept is the vertex. Shapes that do not exhibit a vertex (see Fig. 3.b) may be called *runde Ecke (round corner* or *rounded corner)*. They are constituted by straight line segments smoothly connected by an arc. The line segments can be extended in a straight manner, obeying the Gestalt principle of good continuation, such that they meet in a non-smooth way at a point outside the object's boundary, which can be called a 'virtual vertex' or 'virtual corner point'. This condition of 'non-smooth meeting' is essential, as Fig. 3.e exemplifies: Two arcs that meet smoothly do not constitute a corner. Without the possibility to generate at least a virtual vertex, we cannot ascribe the concept corner.

Summarizing the above considerations, we assume corner to be an idealized concept for which straight lines and vertices play an important role. Therefore the formal characterization we develop in section 4 is done in a geometric framework; by this, we formulate the geometric requirements for the concept specifying the meaning of the lexeme *Ecke*. Nevertheless it is not justified to say that *Ecke* is merely a geometric notion that is part of a specialized mathematical register we learn at school (this might be discussed for lexemes like *parallel* or *angle*). *Ecke* is a natural language term with a common-sense denotation. But probably due to the fact that corner is based on straight lines, vertices, and non-smooth meeting, it is mainly applied to artifacts. Their regular shapes more easily induce straight edges that meet in a non-smooth, sharp manner than the shapes of plants or animals. In addition to *corner* there are a variety of natural language terms for object parts with vertices that are not restricted to straight edges: *apex*, *point*, *thorn*, *horn*, *tip*, etc.

Giving corner a mainly geometric characterization leads to a view corresponding to Pinkal's (1985) general discussion of 'vagueness and precision': Using the lexeme *Ecke* means to neglect some of the properties objects have and to look for properties that are congruent with the geometry of corner. Geometric notions can be meaningfully used in our scope of experience only if they are used flexibly, i.e. allowing some level of imprecision.

3 The Framework of Planar Shape Geometry

This section presents a formal characterization of a geometric framework that is able to specify the spatial constraints discussed so far. The goal is to identify the underlying structures of the spatial concepts corner and kink. The geometric framework is structured similarly to the system presented by Hilbert (1899), which is divided into different groups of axioms. It is developed employing the axiomatic method: An axiomatic system constitutes a system of constraints that determines the properties of basic terms like 'point', '(shape) curve' and 'region' by specifying their interrelations. Since we aim at identifying the general constraints, we develop a description of contours of planar objects that does not require concepts of differential geometry like differentiability, tangents or real numbers. We thereby show that such concepts are not necessary to describe essential shape features. Therefore, a description of shape curves can forgo the use of coordinates or metrical information.[6] The next section presents proposals for characterizing corner and kink as a basis for further discussions.

The geometric structure introduces five types of entities and two primitive relations. The entities are points (denoted by P, Q, R, P', P_1, …), (straight) lines (denoted by l, l', l_1, …), half-planes (denoted by H, H', …), (shape) curves (denoted by c, c_1, …, s, s_1, …, a, a_1, …) and (shape) regions (denoted by *Reg*, *Reg'*, …). Segments and arcs are simple shape curves characterized below. (We use the symbols s, s_1, … and a, a_1, … for reference to simple curves.)[7]

The basic idea is that closed shape curves represent the contours of objects conceptualized as planar, i.e., whose geometric representatives are shape regions. Open shape curves are able to represent linear object conceptualizations as well as trajectories of moving objects (Eisenkolb et al., 1998). Shape curves are constituted by line segments and arcs. Half-planes are introduced in order to distinguish vertices and smooth points of curves. This classification of points will be indispensable in order to characterize the concepts corner and kink.

The primitive relations are the binary relation of incidence (symbolized by ι) and the ternary relation of betweenness for points (symbolized by β). The relation of incidence sets up the relation between points and the other entities and characterizes the fact that a point lies on a line or curve, or in a half-plane or region. The relation of betweenness relates three different points on one line.

3.1 Points and Straight Lines, Incidence and Betweenness

Axioms for Incidence of Points and Straight Lines

Four axioms relate points and straight lines using incidence. Axiom (I1) guarantees that for every line there are at least two different points on it. Axiom (I2) states that any two points lie on one common line. And (I3) says that two lines have at most one point in common. The last axiom (I4) ensures that the underlying structure is at least planar, i.e., not all points are incident with one line.

(I1) $\forall l\, \exists P\, \exists Q \quad [P \neq Q \wedge P\ \iota\ l \wedge Q\ \iota\ l]$

(I2) $\forall P\, \forall Q\, \exists l \quad [P\ \iota\ l \wedge Q\ \iota\ l]$

(I3) $\forall P\, \forall Q\, \forall l_1\, \forall l_2 \quad [P\ \iota\ l_1 \wedge Q\ \iota\ l_1 \wedge P\ \iota\ l_2 \wedge Q\ \iota\ l_2 \Rightarrow (P = Q \vee l_1 = l_2)]$

(I4) $\forall l\, \exists P \quad [\neg(P\ \iota\ l)]$

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[6] The geometric framework we present is closely related to the framework defined by Eschenbach & Kulik (1997), on which an investigation of spatial orderings in the plane and the structures contributed by different frames of reference is based.

[7] As can easily be shown, any of these entities can be represented as a set of points. Since points, in turn, could be represented as sets of lines, half-planes, curves, or regions (cf., Laguna, 1922, Vieu, 1993), any type of entity we consider could be taken as the ontological basis. Since there are several ways of how such a representation (or coding) can be done, we do not assume any of such possibilities as preferable to the others.

Definition (Collinear)

The definitions of collinearity of three points will be useful in the following. For matters of convenience, we define points to be collinear only if they are different. Three (different) points P, Q and R are *collinear*, if they lie on one line.

$$\mathsf{col}(P, Q, R) \quad \Leftrightarrow_{\text{def}} P \neq Q \wedge P \neq R \wedge Q \neq R \wedge \exists l\, [P\ \iota\ l \wedge Q\ \iota\ l \wedge R\ \iota\ l]$$

Axioms for Betweenness of Points

The formula '$\beta(P, Q, R)$' can generally be read as 'Q is between P and R'. We provide seven axioms to specify this relation, six of them describe the order of points on the line. According to axiom (β1), if Q is between P and R, then they are collinear, and consequently distinct. Axiom (β2) expresses the symmetry of betweenness with respect to the first and the third argument: If Q is between P and R, then Q is between R and P. Axiom (β3) ensures that at most one of three points is between the other two. Axiom (β4) states that for three collinear points at least one of them is between the other two, therefore betweenness constitutes a total order. Axiom (β5) secures that if Q is between P and R, and Q' is collinear with them, then Q' is on either side of Q.[8] Axiom (β6) guarantees that lines are unlimited. Axiom (β7) (the axiom of Pasch) specifies an additional constraint on the ordering in the plane according to Fig. 4: If a straight line enters the interior of a triangle, then it also leaves it.

(β1) $\forall P\ \forall Q\ \forall R \quad [\beta(P, Q, R) \Rightarrow \mathsf{col}(P, Q, R)]$

(β2) $\forall P\ \forall Q\ \forall R \quad [\beta(P, Q, R) \Rightarrow \beta(R, Q, P)]$

(β3) $\forall P\ \forall Q\ \forall R \quad [\beta(P, Q, R) \Rightarrow \neg\beta(Q, P, R)]$

(β4) $\forall P\ \forall Q\ \forall R \quad [\mathsf{col}(P, Q, R) \Rightarrow (\beta(P, Q, R) \vee \beta(Q, P, R) \vee \beta(P, R, Q))]$

(β5) $\forall P\ \forall Q\ \forall R\ \forall Q' [\beta(P, Q, R) \wedge \mathsf{col}(Q, Q', P) \Rightarrow (\beta(P, Q, Q') \vee \beta(Q', Q, R))]$

(β6) $\forall P\ \forall Q \quad [P \neq Q \Rightarrow \exists R\, [\beta(P, Q, R)]]$

(β7) $\forall P_1 \forall P_2 \forall P_3 \forall l \quad [\neg(P_1\ \iota\ l) \wedge \neg(P_2\ \iota\ l) \wedge \neg(P_3\ \iota\ l) \wedge \exists Q\, [Q\ \iota\ l \wedge \beta(P_1, Q, P_3)] \Rightarrow$
$\exists R\, [R\ \iota\ l \wedge (\beta(P_1, R, P_2) \vee \beta(P_2, R, P_3))]]$

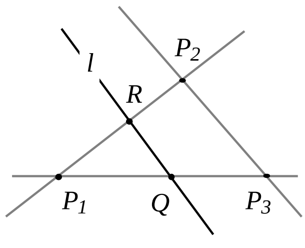

Fig. 4. Illustration of Pasch's axiom (β7)

Pasch's axiom guarantees that the structure we deal with is at most planar. Combined with axiom (I4) this yields that the resulting structure is a plane.

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[8] Huntington (1924) proves the complete independence of this system of five axioms when restricted to one line only.

3.2 Half-Planes in Connection to Points and Straight Lines

The axioms for half-planes are formulated with reference to points and straight lines. They make use of the notion of boundary point.

Definition (Boundary Point)

A point is called *boundary point* of a half-plane, if it is in the half-plane and there is a point outside the half-plane such that all points between them are also outside the half-plane: The boundary point is the last point before leaving the half-plane.

$$\mathsf{bdpt}(H, P) \quad \Leftrightarrow_{\text{def}} P \;\iota\; H \wedge \exists R\, [\neg(R \;\iota\; H) \wedge \forall Q\, [\beta(P, Q, R) \Rightarrow \neg(Q \;\iota\; H)]]$$

Axioms for Half-Planes

Half-planes are bordered by a straight line (H1) and have a point that is not a boundary point (H2). If a point is in a half-plane but not a boundary point, then every point R is in the half-plane, if and only if no point between them is a boundary point (H3). For every straight line and every point there is a half-plane bordered by the line such that the point is in the half-plane (H4). Finally, half-planes can be distinguished by the points that lie in them (H5).

(H1) $\forall H\, \exists l\, \forall P \quad [P \;\iota\; l \Leftrightarrow \mathsf{bdpt}(H, P)]$

(H2) $\forall H\, \exists P \quad [P \;\iota\; H \wedge \neg\mathsf{bdpt}(H, P)]$

(H3) $\forall H\, \forall P \quad [P \;\iota\; H \wedge \neg\mathsf{bdpt}(H, P) \Rightarrow$
$\forall R\, [R \;\iota\; H \Leftrightarrow \forall Q\, [\beta(P, Q, R) \Rightarrow \neg\mathsf{bdpt}(H, Q)]]]$

(H4) $\forall l\, \forall P\, \exists H \quad [\forall Q\, [Q \;\iota\; l \Leftrightarrow \mathsf{bdpt}(H, Q)] \wedge P \;\iota\; H]$

(H5) $\forall H\, \forall H' \quad [\forall P\, [P \;\iota\; H \Leftrightarrow P \;\iota\; H'] \Rightarrow H = H']$

Since every half-plane is bordered by a line and border-lines are uniquely determined, we use the notion $\mathsf{bl}(H)$ to refer to the border-line of half-plane H. As a consequence, half-planes are uniquely determined by their border-line and an additional point on them, and half-planes are convex.

Definition (Border-Line of a Half-Plane)

$$\mathsf{bl}(H) = l \quad \Leftrightarrow_{\text{def}} \forall P\, [P \;\iota\; l \Leftrightarrow \mathsf{bdpt}(H, P)]$$

3.3 Simple and Complex Shape Curves

Curves are usually not described in a geometric framework. But since we need a basis to describe diverse contours of real-world objects, we cannot limit ourselves to straight lines and line segments. Therefore, we introduce shape curves as geometric entities. According to the current context we confine ourselves to curves that do not branch or intersect themselves like the curve of the figure eight. But it will be obvious, how our characterization of shape curves can be generalized.

Curves and points are related by incidence. Arcs and (line) segments are the simplest curves. We proceed by defining segments and arcs first and then presenting the general axioms for curves. On this basis we give a classification of points relative to curves, including a general notion of vertex.

Definition (Enclosed Point of a Curve, Segment, Endpoint of a Segment)

A point is *enclosed* by a curve, if it lies between two points of the curve.

$\mathsf{enc}(c, Q) \qquad \Leftrightarrow_{\text{def}} \exists P\, \exists R\, [P\ \iota\ c \wedge R\ \iota\ c \wedge \beta(P, Q, R)]$

Segments are connected and bounded parts of lines. Thus, they can be described on the basis of betweenness. Any segment has two points such that a point is incident with the segment, exactly if it lies between them or is identical to one of them.

$\mathsf{seg}(s) \qquad \Leftrightarrow_{\text{def}} \exists P\, \exists Q\, \forall R\, [R\ \iota\ s \Leftrightarrow \beta(P, R, Q) \vee P = R \vee Q = R]$

It is convenient to use the notion of *endpoint* in specifying segments. Endpoints are those points of segments that do not lie between any other points of the segment.

$\mathsf{ept}_{\mathsf{seg}}(P, s) \qquad \Leftrightarrow_{\text{def}} P\ \iota\ s \wedge \mathsf{seg}(s) \wedge \neg\mathsf{enc}(s, P)$

The definition for ‘segment’ guarantees that the points of a segment are incident with one line, that segments are convex, and that segments have (exactly) two endpoints. As a consequence of the axiom that curves can be distinguished by the points on them (C11), we will derive that different segments have different pairs of endpoints.

Definition (Supporting Half-Plane, Outer Smooth Point, Outer Vertex)

The description of arcs is more complex. As a preparation we need a way to classify points on curves. The following definition of the *supporting half-plane* and its border-line is the fundamental step towards the classification we need.

Half-plane H *supports* point P with respect to curve c, symbolized by $\mathsf{sup}(H, P, c)$, if P is incident with c and with the border-line of H, and every point incident with c is incident with the half-plane. Border-lines of supporting half-planes are tangents to the curve in this point.

$\mathsf{sup}(H, P, c) \qquad \Leftrightarrow_{\text{def}} P\ \iota\ c \wedge P\ \iota\ \mathsf{bl}(H) \wedge \forall Q\, [Q\ \iota\ c \Rightarrow Q\ \iota\ H]$

We call a point P an *outer smooth point* of curve c, symbolized by $\mathsf{spt_o}(P, c)$, iff it is incident with c and all supporting half-planes of P with respect to c have the same border-line.

$\mathsf{spt_o}(P, c) \qquad \Leftrightarrow_{\text{def}} P\ \iota\ c \wedge \exists H\, [\mathsf{sup}(H, P, c) \wedge \forall H'\, [\mathsf{sup}(H', P, c) \Rightarrow \mathsf{bl}(H) = \mathsf{bl}(H')]]$

If, in contrast, several half-planes with different border-lines support one point P with respect to a curve c, then we call P *outer vertex* of c ($\mathsf{vtx_o}(P, c)$; see Fig. 5).

$\mathsf{vtx_o}(P, c) \qquad \Leftrightarrow_{\text{def}} P\ \iota\ c \wedge \exists H\, \exists H'\, [\mathsf{sup}(H, P, c) \wedge \mathsf{sup}(H', P, c) \wedge \mathsf{bl}(H) \neq \mathsf{bl}(H')]$

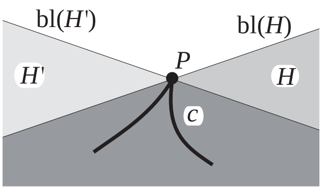

Fig. 5. An outer vertex

In differential geometry uniquely determined tangents ensure that a curve described by real coordinates is differentiable at that point. That we can capture this notion in our more general framework shows that the notion of differentiability and the use of real coordinates is not essential to describe a smooth point.

Since all points of a segment are supported by the half-planes bordered by the line the segment lies on, all points of a segment are outer smooth points or outer vertices and the outer vertices are exactly the endpoints. Line segments are straight. In order to allow curves to be smoothly bent, we introduce arcs. First, we give the complete definition, then we comment the individual clauses of the definition.

Definition (Arc)

An arc is a curve that does not enclose any point on itself, and any point on it is supported by a half-plane. (Therefore, arcs contain only outer smooth points and outer vertices.) An arc has two outer vertices, and one half-plane supports all outer vertices. (This excludes more than two outer vertices.) For any point *R* that does not lie on the arc and is not enclosed by the arc, there is a segment such that the arc is between *R* and the segment in the following sense: any point on the arc is between *R* and some point on the segment and for any point on the segment there is a point on the arc between this point and *R*. More informally stated: The central projection from *R* (the central point) maps the arc bijectively onto the segment (see Fig. 6.c). Therefore the denseness and rectifiability of segments are passed on to arcs.

$$
\begin{aligned}
\mathsf{arc}(a) \quad & \Leftrightarrow_{\mathrm{def}} \forall P\, [P \;\iota\; a \Rightarrow \neg\mathsf{enc}(a, P) \wedge \exists H\, [\mathsf{sup}(H, P, a)]] \wedge \\
& \qquad \exists P\, \exists Q\, [P \neq Q \wedge \mathsf{vtx_o}(P, a) \wedge \mathsf{vtx_o}(Q, a)] \wedge \\
& \qquad \exists H\, [\forall P\, [\mathsf{vtx_o}(P, a) \Rightarrow \mathsf{sup}(H, P, a)]] \wedge \\
& \qquad \forall R\, [\neg(R \;\iota\; a) \wedge \neg\mathsf{enc}(a, R) \Rightarrow \\
& \qquad\quad \exists s\, [\mathsf{seg}(s) \wedge \forall Q\, [Q \;\iota\; s \Rightarrow \exists P\, [P \;\iota\; a \wedge \beta(R, P, Q)]] \\
& \qquad\quad \wedge \forall P\, [P \;\iota\; a \Rightarrow \exists Q\, [Q \;\iota\; s \wedge \beta(R, P, Q)]]]]
\end{aligned}
$$

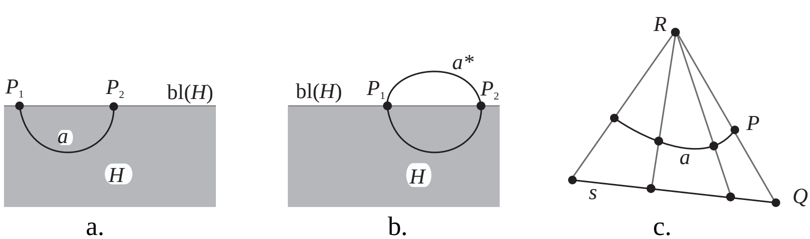

Fig. 6. The definition of arc guarantees a supporting half-plane like *H* in (a) and excludes that a^* in (b) is an arc. (c) is an illustration of the last clause of the definition of arc

Definition (Part of a Curve, Simple Part of a Curve, Endpoint of a Curve, Curves Meeting at an Endpoint, Thin at *P*)

A curve *c'* is *part of* another curve *c* or a *sub-curve* of *c*, if all points of *c'* are incident with *c*.[9]

$$c' \sqsubset c \qquad \Leftrightarrow_{\mathrm{def}} \forall P\, [P \;\iota\; c' \Rightarrow P \;\iota\; c]$$

We call curve *c'* a *simple part* of curve *c* (in symbols $c' \cdot\!\sqsubset c$), if *c'* is a segment or an arc and part of *c*.

$$c' \cdot\!\sqsubset c \qquad \Leftrightarrow_{\mathrm{def}} (\mathsf{seg}(c') \vee \mathsf{arc}(c')) \wedge c' \sqsubset c$$

[9] The sub-curves of an open curve are related according to interval relations as investigated by Knauff et al. (1998).

An endpoint of a curve is on the curve and of any two simple shape curve parts that include it one is part of the other.

$$\mathsf{ept}(P, c) \quad \Leftrightarrow_{\text{def}} P \;\iota\; c \wedge \forall c_1, c_2 \,[c_1 \cdot\!\sqsubset c \wedge c_2 \cdot\!\sqsubset c \wedge P \;\iota\; c_1 \wedge P \;\iota\; c_2 \Rightarrow (c_1 \sqsubset c_2 \vee c_2 \sqsubset c_1)]$$

Two shape curves c, c' *meet* at endpoint P, symbolized by $\mathsf{meet}(P, c, c')$, if P is a common point and all their common points are endpoints. (This allows two curves to meet at both ends.)

$$\mathsf{meet}(P, c, c') \quad \Leftrightarrow_{\text{def}} P \;\iota\; c \wedge P \;\iota\; c' \wedge \forall Q \,[Q \;\iota\; c \wedge Q \;\iota\; c' \Rightarrow \mathsf{ept}(Q, c) \wedge \mathsf{ept}(Q, c')]$$

A curve c is *thin at point* P, if P is on c and there is a line l, such that P is on l and between P and any other point both on c and l there is a point that is not on c.

$$\mathsf{thin}(c, P) \quad \Leftrightarrow_{\text{def}} P \;\iota\; c \wedge \exists l \,[P \;\iota\; l \wedge \forall Q \,[Q \;\iota\; l \wedge Q \;\iota\; c \wedge P \neq Q \Rightarrow \exists R \,[\beta(P, R, Q) \wedge \neg(R \;\iota\; c)]]]$$

Considering segments and arcs we find that they are thin by definition at all their points, and that the outer vertices of segments are their endpoints also in the general sense defined here.

Axioms for Shape Curves

On this basis we can give the collection of axioms for curves. First, we exclude space-filling curves by stating that curves are thin at all their points (C1). This corresponds to Cantor’s definition (cf. Parchomenko 1957).

(C1) $\forall c \,\forall P \quad [P \;\iota\; c \Rightarrow \mathsf{thin}(c, P)]$

Second, all points of a curve are incident with a simple part of that curve (C2), and no point of a curve is the meeting point of three (or more) simple parts of the curve (C3).

(C2) $\forall c \,\forall P \quad [P \;\iota\; c \Rightarrow \exists c' \,[c' \cdot\!\sqsubset c \wedge P \;\iota\; c']]$

(C3) $\forall c_1 \,\forall c_2 \,\forall c_3 \quad [\exists c \,[c_1 \cdot\!\sqsubset c \wedge c_2 \cdot\!\sqsubset c \wedge c_3 \cdot\!\sqsubset c] \Rightarrow \neg\exists P \,[\mathsf{meet}(P, c_1, c_2) \wedge \mathsf{meet}(P, c_1, c_3) \wedge \mathsf{meet}(P, c_2, c_3)]]$

Curves have at most two endpoints (C4) and, if a curve has one endpoint, then it has another one (C5).

(C4) $\forall c \,\forall P \,\forall Q \,\forall R \quad [\mathsf{ept}(P, c) \wedge \mathsf{ept}(Q, c) \wedge \mathsf{ept}(R, c) \Rightarrow (P = Q \vee P = R \vee Q = R)]$

(C5) $\forall c \,\forall P \quad [\mathsf{ept}(P, c) \Rightarrow \exists Q \,[\mathsf{ept}(Q, c) \wedge P \neq Q]$

If two curves meet at one endpoint, then there is a curve that has exactly the points of the two given curves (C6). On the other hand, if a curve c_1 is part of another curve c, then there is an additional sub-curve c_2 of c that meets c_1 (C7). This secures that curves are (path-)connected.

(C6) $\forall c_1 \,\forall c_2 \quad [\exists P \,[\mathsf{meet}(P, c_1, c_2)] \Rightarrow \exists c \,\forall Q \,[Q \;\iota\; c \Leftrightarrow (Q \;\iota\; c_1 \vee Q \;\iota\; c_2)]$

(C7) $\forall c \,\forall c_1 \quad [c_1 \sqsubset c \wedge c \neq c_1 \Rightarrow \exists c_2 \,\exists P \,[c_2 \sqsubset c \wedge \mathsf{meet}(P, c_1, c_2)]]$

Axiom (C8) secures that every curve is constituted by finitely many simple curves. We have to notice that this formulation needs quantification over natural numbers.

(C8) $\forall c \,\exists n \,\exists c_1 \ldots \exists c_n [c_1 \cdot\!\sqsubset c \wedge \ldots \wedge c_n \cdot\!\sqsubset c \wedge \forall P \,[P \;\iota\; c \Rightarrow (P \;\iota\; c_1 \vee \ldots \vee P \;\iota\; c_n)]]$

As an additional axiom we assume that any two points define a segment such that they are its endpoints (C9).

(C9) $\forall P \,\forall Q \quad [P \neq Q \Rightarrow \exists s \,[\mathsf{seg}(s) \wedge \mathsf{ept}_{\mathsf{seg}}(P, s) \wedge \mathsf{ept}_{\mathsf{seg}}(Q, s)]]$

Axiom (C10) states that for every arc *a* and two points on it there is an arc that has exactly those points of *a* that lie in a half-plane bordered by a line through the two points. (One simple result is that the outer vertices of arcs are their (only) endpoints.)

$$\text{(C10)}\ \forall a\ \forall P\ \forall Q \qquad [\mathsf{arc}(a) \wedge P \neq Q \wedge P\ \iota\ a \wedge Q\ \iota\ a \Rightarrow \exists a'\exists H\ [\mathsf{arc}(a') \wedge P\ \iota\ \mathsf{bl}(H) \wedge Q\ \iota\ \mathsf{bl}(H) \wedge \forall R\ [R\ \iota\ a' \Leftrightarrow R\ \iota\ a \wedge R\ \iota\ H]]]$$

Finally, curves differ in the points they are incident with (C11). Therefore, curves can be represented as sets of points, although we do not employ such a representation.

$$\text{(C11)}\ \forall c\ \forall c' \qquad [\forall P\ [P\ \iota\ c \Leftrightarrow P\ \iota\ c'] \Rightarrow c = c']$$

Definition (Vertex of a Curve, Smooth Point of a Curve, Turning Point of a Curve, Open Shape Curve, Sum of Two Meeting Curves)

We call a point *P* a *smooth point* of a curve *c*, symbolized by $\mathsf{spt}(P, c)$, if it is an outer smooth point of a sub-curve of *c*.

$$\mathsf{spt}(P, c) \quad \Leftrightarrow_{\text{def}}\ P\ \iota\ c \wedge \exists c'\,[c' \sqsubset c \wedge \mathsf{spt_o}(P, c')]$$

If a point *P* is an outer vertex of a sub-curve of curve *c* and not an endpoint of this sub-curve, then it is an *inner vertex* of *c*, symbolized by $\mathsf{vtx}(P, c)$ (see Fig. 7).

$$\mathsf{vtx}(P, c) \quad \Leftrightarrow_{\text{def}}\ P\ \iota\ c \wedge \exists c'\,[c' \sqsubset c \wedge \neg\mathsf{ept}(P, c') \wedge \mathsf{vtx_o}(P, c')]$$

A point *P* is called *turning point* of *c* (symbolized as $\mathsf{tpt}(P, c)$), if it is an endpoint of every sub-curve *c'* such that *P* is supported by a half-plane with respect to *c'*.

$$\mathsf{tpt}(P, c) \quad \Leftrightarrow_{\text{def}}\ P\ \iota\ c \wedge \neg\mathsf{ept}(P, c) \wedge \forall c'\,[c' \sqsubset c \wedge \exists H\ [\mathsf{sup}(H, P, c')] \Rightarrow \mathsf{ept}(P, c')]]$$

It is possible to show that every point on a curve that is not an endpoint of this curve belongs to exactly one of the three classes: The point is either a smooth point or a turning point or an inner vertex.

If a shape curve *c* does not have an endpoint, then we call the shape curve *closed* (in symbols: $\mathsf{cl}(c)$). Otherwise we call the shape curve *open*.

$$\mathsf{cl}(c) \quad \Leftrightarrow_{\text{def}}\ \neg\exists P\ [\mathsf{ept}(P, c)]$$

If two shape curves c_1 and c_2 meet, then we call the curve *c* that has exactly the points of c_1 and c_2 (see C6 and C11) their *sum* (symbolized as $c = c_1 \sqcup c_2$).

$$c = c_1 \sqcup c_2 \ \Leftrightarrow_{\text{def}}\ \forall Q\ [Q\ \iota\ c \Leftrightarrow (Q\ \iota\ c_1 \vee Q\ \iota\ c_2)]$$

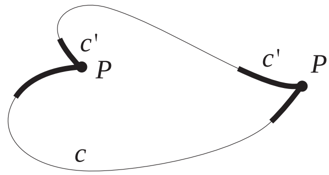

Fig. 7. A curve with two inner vertices

3.4 Shape Regions

The characterization of shape regions we present here is not meant to be a worked out theory of regions in general as, e.g., underlying the calculus investigated by Renz &

Nebel (1998). It just needs to fit certain purposes for describing corners. The most important aspect for this purpose is that regions are bounded by closed curves. One consequence is that they are connected and have no holes.

Definition (Part of a Region, Boundary Point of a Region, Convex)

A region *Reg'* is *part of* another region *Reg* or a *sub-region* of *Reg*, if all points of *Reg'* are incident with *Reg*.

$$Reg' \sqsubset Reg \quad \Leftrightarrow_{\text{def}} \forall P\, [P\ \iota\ Reg' \Rightarrow P\ \iota\ Reg]$$

A point is *a boundary point* of a region *Reg*, if it is in *Reg* and any point lying between it and a point not in *Reg* is also not in *Reg*.

$$\mathsf{bdpt}(Reg, P) \quad \Leftrightarrow_{\text{def}} P\ \iota\ Reg \wedge \exists R\, [\neg(R\ \iota\ Reg) \wedge \forall Q\, [\beta(P, Q, R) \Rightarrow \neg(Q\ \iota\ Reg)]]$$

A region is convex, if every point between two points of the region is in the region.

$$\mathsf{cvx}(Reg) \quad \Leftrightarrow_{\text{def}} \forall Q\, [\exists P\, \exists R\, [P\ \iota\ Reg \wedge R\ \iota\ Reg \wedge \beta(P, Q, R)] \Rightarrow Q\ \iota\ Reg]$$

Axioms for (Shape) Regions

The axioms for shape regions state that every region has a boundary that is a closed curve (R1), that it includes a point that is not a boundary point (R2), that between any point (properly) in the region and any point outside the region there is a boundary point of the region (R3), that shape regions do not contain any straight line completely (R4), and that two regions are distinguished by the points in them (R5).

(R1) $\forall Reg\, \exists c \quad [\mathsf{cl}(c) \wedge \forall P\, [P\ \iota\ c \Leftrightarrow \mathsf{bdpt}(Reg, P)]]$

(R2) $\forall Reg\, \exists P \quad [P\ \iota\ Reg \wedge \neg\mathsf{bdpt}(Reg, P)]$

(R3) $\forall Reg\, \forall P \quad [P\ \iota\ Reg \wedge \neg\mathsf{bdpt}(Reg, P) \Rightarrow$
$\quad\quad \forall R\, [\neg(R\ \iota\ Reg) \Rightarrow \exists Q\, [\beta(P, Q, R) \wedge \mathsf{bdpt}(Reg, Q)]]]$

(R4) $\forall Reg\, \forall l\, \exists P \quad [P\ \iota\ l \wedge \neg(P\ \iota\ Reg)]$

(R5) $\forall Reg\, \forall Reg' \quad [\forall P\, [P\ \iota\ Reg \Leftrightarrow P\ \iota\ Reg'] \Rightarrow Reg = Reg']$

Axiom (R4) guarantees that the complement of a region is not a region itself. Since every shape region has a uniquely determined boundary curve, we use the notion $\mathsf{bd}(Reg)$ to refer to the boundary of region *Reg*.

Definition (Boundary of a Region)

$$\mathsf{bd}(Reg) = c \quad \Leftrightarrow_{\text{def}} \forall P\, [P\ \iota\ c \Leftrightarrow \mathsf{bdpt}(Reg, P)]$$

The geometric framework developed in this section is not specialized to the concepts corner and kink that we investigate in the present paper. It can be applied to any contour or planar drawing of a physical object without holes. Since it is formulated without reference to coordinates and differentiability, we have shown that formal representation and characterization of shape features does not presuppose the use of differential geometry. Additionally, the geometric system does not introduce or measure angles. As a consequence, the framework is not strong enough to specify orthogonality, but could be enriched by a group of congruence axioms, if necessary. In the next section we show that this general framework is sufficient to characterize the geometric aspects of corner and kink.

4 Characterizations of Corner and Kink

The framework of planar shape geometry forms the basis for the formal characterizations of corner and kink. Based on the geometric constellation described by these concepts there are several alternatives to select one of the entities involved as the referent of the corresponding nouns. To give an impression of the general spectrum of possibilities and to show the independence of this problem from the geometric specification, we propose a selection of alternatives. The linguistic analysis in the next section discusses these characterizations in order to develop criteria to select among them.

To refer to the mapping of objects to their spatial conceptualization (see section 2), we employ a function named loc. Objects are referred to by O and O'. We use the notion 'linear(loc(O))' to state that the representation of object O is a shape curve and 'planar(loc(O))' to state that it is a shape region.

$$\mathsf{loc}\text{: objects} \rightarrow \text{shape regions} \cup \text{shape curves}$$

4.1 Characterizations of Corner

We give five alternatives for the characterizations of corner. They differ with respect to the entity they directly specify: The referent can be a point, a boundary, a region or an object part. In addition, two characteristic regions are discussed as referents of *Ecke*. They agree in assuming the object in question to be planar and in their specification of the basic geometric structure. All characterizations share the geometric characteristics that an inner vertex and a part of the object's boundary that is constituted by two segments meeting at the vertex are involved.

Corners as Points: Corner$_{pt}$

The first characterization (named 'corner$_{pt}$') focuses on the vertex of the boundary of the object as the geometric referent of the noun *Ecke*. It says that the corner P of a planar object O is a vertex of O's boundary given by two straight boundary parts (s_1 and s_2) that meet non-smoothly.

$$\mathsf{corner_{pt}}(P, O) \quad \Leftrightarrow_{def} \quad \mathsf{planar}(\mathsf{loc}(O)) \wedge \mathsf{vtx}(P, \mathsf{bd}(\mathsf{loc}(O))) \wedge \exists s_1 \exists s_2 [\mathsf{seg}(s_1) \wedge \mathsf{seg}(s_2) \wedge \mathsf{meet}(P, s_1, s_2) \wedge s_1 \sqsubset \mathsf{bd}(\mathsf{loc}(O)) \wedge s_2 \sqsubset \mathsf{bd}(\mathsf{loc}(O))]$$

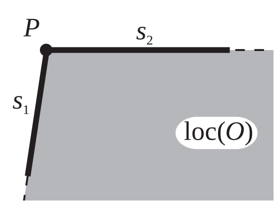

Fig. 8. Depiction of a corner for the characterizations of corner$_{pt}$ and corner$_c$

Corners as Curves: Corner$_c$

The characterization named 'corner$_c$' singles out the boundary part formed by the two meeting segments. It says that a corner c of an object O is a part of the boundary of O

that is constituted by two straight boundary parts (s_1 and s_2) that meet in a vertex P. Thus, the geometric referent of the noun *Ecke* is an open curve. This specification is indefinite concerning the extent of the denoted sub-curve. It does not include any assumption concerning the length of the constituting segments. This reflects the indefiniteness of a corner's size discussed in section 2.

$$\mathsf{corner_c}(c, O) \quad \Leftrightarrow_{def} \quad \mathsf{planar}(\mathsf{loc}(O)) \wedge c \sqsubset \mathsf{bd}(\mathsf{loc}(O)) \wedge \exists P\, [\mathsf{vtx}(P, \mathsf{bd}(\mathsf{loc}(O))) \wedge \exists s_1\, \exists s_2\, [\mathsf{seg}(s_1) \wedge \mathsf{seg}(s_2) \wedge \mathsf{meet}(P, s_1, s_2) \wedge c = s_1 \sqcup s_2]]$$

Corners as Regions: $\mathsf{Corner_{Reg}}$

In the third characterization we shift the focus to the region included by the boundary part specified by $\mathsf{corner_c}$. It states that the corner *Reg* of object *O* is a convex sub-region of the object's region, such that the two regions share a boundary part constituted by straight segments (s_1 and s_2) that meet in a vertex P. This specification is neutral concerning the extent and the exact shape of the region: it can be the triangle or a rectangle enclosed by the boundary part, or some other convex region. (Based on evidence for or against some shape, this characterization could of course be refined.)

$$\begin{aligned}\mathsf{corner_{Reg}}(Reg, O) \quad \Leftrightarrow_{def} \quad & \mathsf{planar}(\mathsf{loc}(O)) \wedge Reg \sqsubset \mathsf{loc}(O) \wedge \mathsf{cvx}(Reg) \wedge \\ & \exists P\, [\mathsf{vtx}(P, \mathsf{bd}(\mathsf{loc}(O))) \wedge \\ & \exists s_1\, \exists s_2\, [\mathsf{seg}(s_1) \wedge \mathsf{seg}(s_2) \wedge \mathsf{meet}(P, s_1, s_2) \wedge \\ & \forall Q\, [Q\ \iota\ s_1 \sqcup s_2 \Leftrightarrow \mathsf{bdpt}(\mathsf{loc}(O), Q) \wedge \mathsf{bdpt}(Reg, Q)]]]\end{aligned}$$

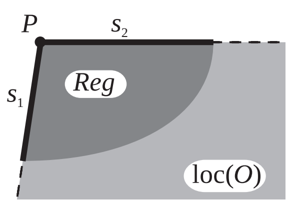

Fig. 9. Depiction of a corner for characterizations of $\mathsf{corner_{Reg}}$ and $\mathsf{corner_O}$

Corners as Object Parts: $\mathsf{Corner_O}$

The fourth characterization named '$\mathsf{corner_O}$' is analogous to $\mathsf{corner_{Reg}}$ except that not the region but an object part occupying the region is singled out. This reflects the intuition that corners are parts of objects. The region of the object part ($\mathsf{loc}(O')$) has to fulfill the same conditions as the region in the characterization of $\mathsf{corner_{Reg}}$.

$$\begin{aligned}\mathsf{corner_O}(O', O) \quad \Leftrightarrow_{def} \quad & \mathsf{planar}(\mathsf{loc}(O)) \wedge \mathsf{loc}(O') \sqsubset \mathsf{loc}(O) \wedge \mathsf{cvx}(\mathsf{loc}(O')) \wedge \\ & \exists P\, [\mathsf{vtx}(P, \mathsf{bd}(\mathsf{loc}(O))) \wedge \\ & \exists s_1\, \exists s_2\, [\mathsf{seg}(s_1) \wedge \mathsf{seg}(s_2) \wedge \mathsf{meet}(P, s_1, s_2) \wedge \\ & \forall Q\, [Q\ \iota\ s_1 \sqcup s_2 \Leftrightarrow \mathsf{bdpt}(\mathsf{loc}(O), Q) \wedge \mathsf{bdpt}(\mathsf{loc}(O'), Q)]]]\end{aligned}$$

Corners as Internally Structured Regions: $\mathsf{Corner_{sReg}}$

In contrast to the characterizations above, $\mathsf{corner_{sReg}}$ specifies a corner of an object *O* as an internally structured region *Reg* (see Fig. 10). It is modeled as a convex region again, though this region is not completely included in the object's region, but intersects with it. Its internal structure includes the vertex *P* and two segments (s_1 and s_2)

that are part of the boundary of the object's region and meet at *P*. This characterization treats the convex and the concave region parts around the vertex as more symmetric than $\mathsf{corner}_{\mathsf{Reg}}$.

$$\mathsf{corner}_{\mathsf{sReg}}(Reg, O) \Leftrightarrow_{\text{def}} \mathsf{planar}(\mathsf{loc}(O)) \wedge \mathsf{cvx}(Reg) \wedge \exists P\, [\mathsf{vtx}(P, \mathsf{bd}(\mathsf{loc}(O))) \wedge \exists s_1\, \exists s_2\, [\mathsf{seg}(s_1) \wedge \mathsf{seg}(s_2) \wedge \mathsf{meet}(P, s_1, s_2) \wedge \forall Q\, [Q\ \iota\ s_1 \sqcup s_2 \Leftrightarrow \mathsf{bdpt}(\mathsf{loc}(O), Q) \wedge Q\ \iota\ Reg]]]$$

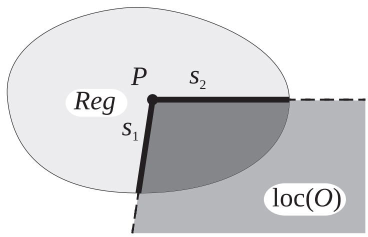

Fig. 10. Depiction of a corner for the characterization of $\mathsf{corner}_{\mathsf{sReg}}$

4.2 Characterizations of Kink

Since kinks are ascribed to linear objects, they can be treated in a manner similar to the first or second characterization of corner. But there are two main differences for the characterization of kink: First of all, the object itself and not its boundary specifies the underlying curve. Secondly, the simple curve parts that form the vertex of a kink need not to be straight. The corresponding sub-curves can be arcs as well.

Kinks as Points: $\mathsf{Kink}_{\mathsf{pt}}$

In the characterization called '$\mathsf{kink}_{\mathsf{pt}}$', a kink is just a vertex of a linear object.

$$\mathsf{kink}_{\mathsf{pt}}(P, O) \Leftrightarrow_{\text{def}} \mathsf{linear}(\mathsf{loc}(O)) \wedge \mathsf{vtx}(P, \mathsf{loc}(O))$$

Kinks as Curve Parts: $\mathsf{Kink}_{\mathsf{c}}$

In the second characterization, a kink is a part of the curve including exactly one vertex. The curve is constituted by two simple curves meeting in this vertex.

$$\mathsf{kink}_{\mathsf{c}}(c, O) \Leftrightarrow_{\text{def}} \mathsf{linear}(\mathsf{loc}(O)) \wedge c \sqsubset \mathsf{loc}(O) \wedge \exists P\, [\mathsf{vtx}(P, c) \wedge \exists c_1\, \exists c_2\, [(\mathsf{seg}(c_1) \vee \mathsf{arc}(c_1)) \wedge (\mathsf{seg}(c_2) \vee \mathsf{arc}(c_2)) \wedge \mathsf{meet}(P, c_1, c_2) \wedge c = c_1 \sqcup c_2]]$$

The axiomatic characterizations given in this section form the basis for our further discussion of the semantics of *Ecke* as one example of shape nouns. Based on its interaction with spatial prepositions we aim at selecting among the alternatives the proper conceptual counterpart by answering the question, what kind of entities are denoted by *Ecke*?

5 An Analysis of *Ecke*

Having given proposals for the geometric characterization of the concepts corner and kink, we now return to the relation between concepts and lexemes as discussed in section 1 (see Fig. 1). Whereas the axiomatic method is able to yield a variety of conceptual designs for the semantics of shape nouns, the linguistic analysis shall give criteria to choose among them. The aim of discussing the interpretations of *Ecke* in different linguistic contexts is to extract the most appropriate conceptual structure as the semantics of the lexeme.

The notion 'shape noun' reflects the idea that these nouns encode shape features of objects. The referent of the relational noun *Ecke* is established on the basis of the shape of a complex object like a room or a sheet of paper. Shape properties are spatial properties of extended objects, i.e. objects that properly occupy some area. In lexical semantics spatial properties are mainly investigated in relation to prepositions: Topological prepositions like *in, on* or *at* spatially relate two objects and thereby impose requirements on the spatial properties of these objects. This is made explicit by Herskovits (1986), and–for German *in*–by Pribbenow (1993) and Buschbeck-Wolff (1994). Therefore, the analysis of the shape noun *Ecke* in relation to topological prepositions should give us insights concerning the spatial properties of its referents. Our discussion is based on prepositional phrases containing the topological prepositions *in (in)* and *an (at, on),* and *Ecke* as the head noun of their internal argument. We start by considering the German preposition *in*.

As assumed by Bierwisch (1988), Klein (1991), Herweg (1989) and Wunderlich & Herweg (1991), the semantics of *in* is roughly that an object is localized in the interior of another object; thus *in* yields the relation of local containment. This containment may be realized in different ways; i.e., the localized object being in the interior of a hollow object (*der Stuhl im Zimmer / the chair in the room*) or being contained in a solid object (*der Splitter im Finger / the splinter in the finger*). Leaving details aside, one object is localized *in* another object, the so-called 'reference object', if and only if the reference object supplies a spatial container for the localized object. Following this line, (4) leads to the demand that *Ecke* has to supply a region in which chairs or spots can be contained.[10]

(4) a. der Stuhl in der Ecke des Zimmers	(the chair in the corner of the room)
b. der Fleck in der Ecke des Teppichs	(the spot in the corner of the carpet)
c. der Kratzer in der Ecke des Fensters	(the scratch in the corner of the window)
d. der Riß in der Ecke des Papiers	(the tear in the corner of the sheet)

[10] Note that this requirement of supplying a region only holds, if the contained object is two- or three-dimensional. Points and linear objects like kinks can be contained in linearly conceptualized objects like sticks as well (as in *der Knick in dem Stab (the kink in the stick*)).

In (4.b–d) the localized object is contained in a planar region with a vertex and two boundary parts of the objects as boundaries of the corner. In (4.a), a three-dimensional corner provides the region for the localized three-dimensional object, the chair.[11]

As *in* demands a region, the formal characterizations of *Ecke* to denote a point (corner_{pt}) or a curve (corner_{c}) seem to be less plausible. Instead, *Ecke* should be captured either as denoting a region, or an object part that, like other material objects that can occur as reference objects, has to be mapped onto its IN-region, as suggested in Herskovits (1986). But based on (4) we can exclude one of the region-based characterizations of *Ecke*, namely corner_{sReg}. *In der Ecke* cannot specify a region outside the underlying object (e.g., the room). Since the region corner_{sReg} spreads outside the object, it does not qualify as the referent of *Ecke.*

Now let us turn to another topological preposition, *an (at).* Following Herweg (1989), *an* means 'in the region proximal to a reference object'.[12] Obviously, the proximal region that *an* might bring about varies if the corner is a vertex, a curve, an object part or a region. Consider the following examples:

(5) a. Der Kiosk ist an der Ecke.
(The kiosk is at the corner.)
b. Die Spinne sitzt an der Ecke des Fensters.
(The spider is sitting at the corner of the window.)

The interpretation of the examples in (5) allows the localized objects to be inside or outside the corner region as long as they are close enough. In (5.a), the kiosk is nearby a corner, e.g., the corner constituted by a crossing or the corner of a house. It can even be inside the house with the corner. Accordingly, (5.b) does not inform us whether the spider is sitting on the outer edge of the window frame or just *in the corner*, i.e. on the window. Thus, *an der Ecke* allows the localized object to be in the exterior as well as in the interior of the corner with respect to the reference object denoted by *Ecke*.

In contrast to the results for the preposition *in,* combinations with *an* suggest a more symmetric treatment of the inside and the outside of corners. This prefers the formal characterizations of corner_{pt} and corner_{c} above corner_{Reg} and corner_{O}.

This observation can have consequences for the semantics of the topological prepositions we employed for analyzing *Ecke*: In connection with nouns denoting concrete objects with clear boundaries, *an* and *in* exclude each other. In other words, if an object is in a reference object then it is not at (*an*) the reference object, and vice versa. We found that this dependence does not exist for *Ecke*. *An der Ecke* and *in der Ecke* may denote overlapping regions. Thus, there is no mutual exclusion of the

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[11] Examples like *der Stuhl in der Ecke des Teppichs (the chair in the corner of the carpet)* can be seen as evidence that an analysis based on planar conceptualizations would also be sufficient for (4.a), as in this example the relevant containing region is planar.

[12] Herweg (1989) contrasts *an* with *bei (near, by)*, which, according to his analysis, additionally specifies the condition 'not being in contact with the reference object'. Thus, *an* allows contact, but does not require it, whereas *bei* excludes contact. Other authors assume that *an* requires contact and *bei* excludes contact. Note that the difference 'allowing contact' vs. 'requiring contact' for the semantics of *an* is not essential for the following.

outside of an object (which is relevant for *an*) and the inside of an object (being relevant for *in*).

One way to handle this may be to analyze *Ecke* on the conceptual level as corner_c and *in* and *an* as mapping this partial contour to regions based on convexity and closeness and contact, respectively. But another consequence might be to have another look at the shape dependencies of *in* and *an* by further analyses of combinations with other shape nouns.

The following examples give further evidence that the choice we planed to make is not clearly decided by the linguistic behavior of *Ecke*. They emphasize the role of the vertex point in the concept of corner. They give evidence that the reference to the vertex can be regarded as essential for the linguistic behavior of *Ecke*.

Considering (6.a), the cupboard has to be localized as close as possible to the wall to make the sentence true. If *genau (exactly)* is omitted, the cupboard might also be standing further away from the wall; *an* thus allows for a proximal region which is not restricted to the closest region. But *genau* restricts the possible ranges of locations for the localized object. *Genau* therefore serves as a test to determine where the exact AN-region is to find, and, in doing so, it indicates which part of the reference object this region has to be close to. In (6.a), it is the surface of the wall.

(6) a. Der Schrank steht genau an der Wand.
(The cupboard is standing exactly at the wall.)
b. Die Spinne sitzt genau an der Ecke des Fensters.
(The spider is sitting exactly at the corner of the window.)

The interpretation of (6.b), is that the spider is sitting as close as possible to the vertex. Hence, we are led to conclude that corner_{pt} is the most plausible characterization of *Ecke* in this context. Example (7) offers additional evidence in this direction.

(7) Der Teppich reicht bis an die Ecke des Zimmers.
(The carpet reaches the corner of the room.)

In (7), *reichen bis* (*to reach*) uses the directional version of *an* (obvious from the accusative case of *die Ecke* in contrast to the dative case of *der Ecke* in, e.g., (6.b)). The interpretation we derive is that some edge (or corner) of the carpet is as close as possible to the vertex of the room's corner. Thus, if exactness is emphasized in connection to *Ecke*, closeness (or contact) to the vertex is the preferred interpretation.

Concerning the axiomatic characterization this seems to suggest that *Ecke* refers to the vertex. We even might conclude that corner_{Reg} is inadequate since being in the region is not enough for being exactly at the corner. On the other hand, assuming the vertex to be the referent of *Ecke* leads to, as stated above, more complex explanations for combinations with *in*.

Summarizing the above we find that *Ecke* does not behave as nouns denoting concrete objects do. In connection with *in* and *an* we see that *Ecke* does not clearly distinguish between interior and exterior. In connection with *in*, *Ecke* seems to refer to a region or an object part, and the conceptualization as a vertex is excluded. But the preposition *an* seems to require direct access to the vertex.

We conclude that *Ecke* has a variable denotation within a conceptual spectrum based on a stable geometric characterization. Hence, it behaves like a polysemous word, comparable to institution words like *school, university,* or *government* investigated by Bierwisch (1983). These words offer systematic alternations, including the readings *institution* and *building* (compare *The school bored him to death* and *The school is burning*). These readings correspond to different conceptual variants, comparable to the variants we found for *Ecke.*

In this paper, we have offered five options for conceptually different referents of *Ecke*. We only exclude one of them on the basis of linguistic considerations, namely corner_{sReg}, but found evidence for all the others. Thus the spectrum still ranges from a vertex (corner_{pt}) via a curve or a region (corner_{c}, corner_{Reg}) to an object part (corner_{O}).

6 Conclusion

The present paper offers a formal, geometric approach to describe shape and specify shape concepts, which are part of our spatial knowledge that underlies the semantics of natural language expressions and that diverse cognitive abilities like object recognition and haptic or auditory perception are based on. This characterization is based on features of the contour of an object, rather than its axes.

The formal description is formulated in planar shape geometry developed in this paper as well. Employing the axiomatic method it enriches the basic inventory of planar ordering geometry by shape curves. In this framework we are able to dispense with the use of coordinates and limits, thus showing that a formal description of curves–as needed for the current purpose of describing contours–does not require the notions of differential geometry. Planar shape geometry supplies a general inventory for the description of contours of planar objects without holes. Thus, we offer a tool for future research on contour information, especially concepts that make use of vertices, e.g., concepts related to lexemes like *apex, point, thorn, horn, tip,* etc.

The linguistic analysis of the German nouns *Ecke* and *Knick* sheds light on the underlying concepts corner and kink. It yields the result that the concepts kink and corner primarily differ in dimensionality information. In addition, we found that the behavior of *Ecke* in combination with the topological prepositions *in* and *an* cannot systematically be explained on the basis of only one characterization for corner. Of the five alternative characterizations we offered, we only found evidence to exclude one. This also suggests that the case of analyzing *in* and *an* has to be reopened, especially in combination with shape nouns in addition to nouns denoting concrete material and bounded objects.

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